The Sine Wave

The sine wave and how it works — dissecting and reassembling it with vectors.

By Paul Chappell

There are a number of ways of representing a sine wave. **Fig. 1** shows two possibilities. The first is the usual graphical representation. A diagram like this gives a good deal of information about the wave: its shape, amplitude, frequency and phase at the chosen starting point, just about all you could ever want to know about it.

Fig.1b shows a more abstract representation of the same wave. The frequency is shown by the position of the line along the horizontal scale and amplitude is given by the height of the line. This diagram is more concise than Fig. 1a (and takes less drawing skill) but some of the information is lost. The shape of the wave is not shown, so to interpret the diagram you have to know what a sine wave looks like. Another loss is phase information. There's also a slightly less obvious loss — see if you can spot it. (Look for a way to alter Fig. 1a to give another wave which would have the same representation in Fig. 1b).

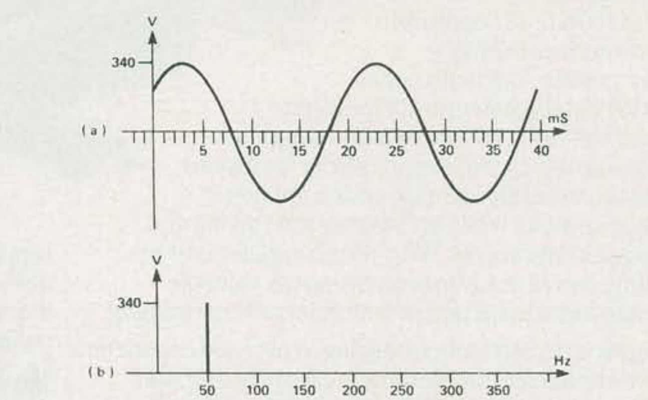

Fig. 1(a). The normal view of a sine wave, and (b) an alternative representation.

The loss of phase information means that if two or more sine waves are shown on the same diagram, it is impossible to say exactly what time domain waveform they represent. **Fig. 2a** may represent either Fig. 2b or Fig. 2c or something else entirely. Without knowing the relative phases of the two sine waves, there is no way to decide. (Strictly speaking it makes no sense to speak of the phase difference between two waves of different frequencies since it changes at every instant. What you can do is to compare each component with a sine or cosine of its own frequency at a particular instant in time which will give enough information to decide the time relationship of the two waves.

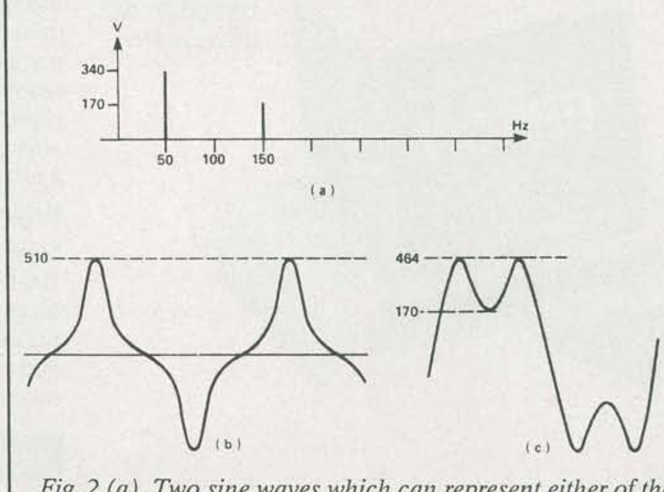

Fig. 2 (a). Two sine waves which can represent either of the waves (b) or (c).

For example, if both waves were exactly in phase with the sine of their own frequency at some instant in time, Fig. 2c would be the correct time domain waveform. On the same pedantic note, I'd better explain that I'm using the phrase "sine wave" to mean "any wave which is sinusoidal in shape" and unadorned "sine" and "cosine' to mean waves that begin at 0 and 1 respectively at some instant $t=0$. Phase relationships are difficult to talk about without sounding too text-booky and I don't want to confuse you in my attempts to do so.

In many situations the frequency spectrum of a wave form is the most important characteristic. An example is frequency interlacing in colour TV systems. The frequency spectrum of a monochrome TV signal has the strongest frequency components at multiples of the line frequency on either side. **Fig. 3a** is an idealized diagram of a portion of this spectrum. The gaps in the spectrum allow a rather clever trick to reduce the bandwidth needed for colour TV transmission. Instead of avoiding interference by putting the colour subcarrier at some frequency way above the monochrome information, it can be slotted in to fill up the gaps.

Suppose the colour subcarrier frequency was chosen to be 200.5 times the line frequency. The main additional frequencies introduced would be the subcarrier and its sidebands separated by multiples of the line frequency, which neatly fills the gaps left by the monochrome spectrum (**Fig.3b**). The half-line offset means that the subcarrier frequency will cancel out to some extent on successive pictures and too high an amplitude would still cause some very unpleasant effects.

The moral is that it is very important to keep an eye on the practical interpretation of abstract information. The offset used in the European PAL system, by the way, is 1/4 line rather than 1/2 line; this simple explanation leaves out some other rather significant considerations which mean that 1/2 line offset is not the best choice, but the principle still applies.

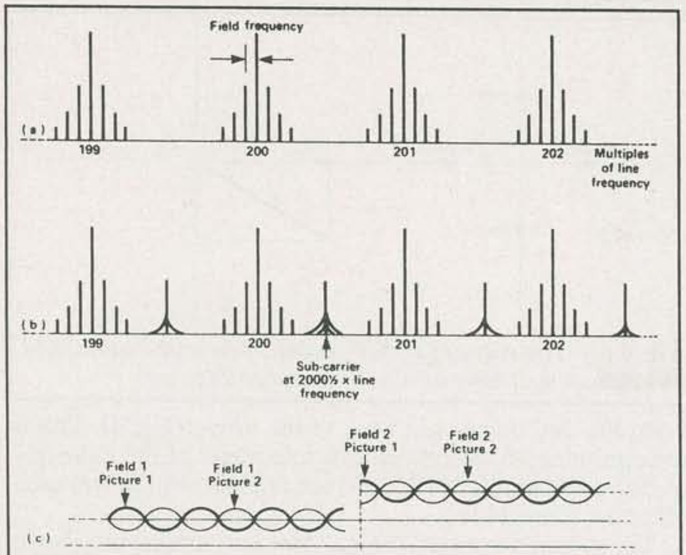

Fig. 3 (a). A portion of the PAL TV spectrum (mono), (b) the colour subcarrier slotted in the gaps, and (c) the effect on the mono picture.

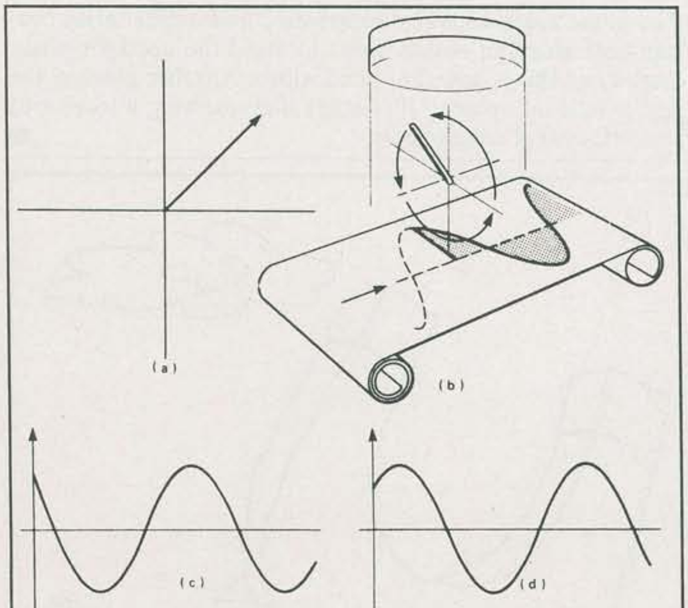

Fig. 4 (a) A vector, (b) the wave-drawing machine, (c) a counterclockwise wave and (d) a clockwise wave.

Yet another representation of a sine wave is shown in **Fig. 4a**, and it's this one I really want to concentrate on. If you haven't seen this kind of diagram before, it will take a little imagination to see how it works. Just suppose for a moment that the arrow on the diagram is actually a piece of wood baton pivoted at the origin, which its free end moving steadily in an counterclockwise circle. Now suppose that there's a spotlight above it and a screen below. As the baton moves, the shadow on the screen will shorten until the rod becomes vertical, then lengthen again in the opposite direction, then shorten again until the rod is pointing downwards, and so on.

Now suppose that instead of a screen, the shadow is cast on a strip of photographic material that is exposed by darkness (don't ask me where you can buy it). The photographic strip is moved along by the same invisible motor which is turning the baton (I said you'd need a good imagination.). When the paper is developed, it will have a trace on it as **Fig.4b**. A Sine wave!

A complete cycle of the sine wave corresponds exactly to one rotation of the baton. The amplitude will be equal to the longest shadow cast by the baton which will happen when it is horizontal and so will be equal to its length. The phase relative to a cosine will be equal to the angle the baton makes with the horizontal when the machine is started up. In other words, if I gave you a photograph (or a diagram) of the initial position of the baton and told you the speed of rotation, you could predict exactly what sine wave the machine would draw. That brings us back to Fig.4a.

The direction of rotation of sine-wave drawing machines is important. Figure 4a could draw either Fig. 4c or Fig. 4d depending on which way it was turned. By convention, sine wave drawing machines always turn counterclockwise, so Fig.4c is correct.

Diagrams like Fig.4a are called vectors. The most significant loss of information here is frequency; I have to tell you how fast the rod is rotating. This may seen like rather an important piece of information to lose but often it can be assumed from the context. At other times it can be useful to know how a circuit responds to a certain frequency. In all these cases, frequency is part of the background information

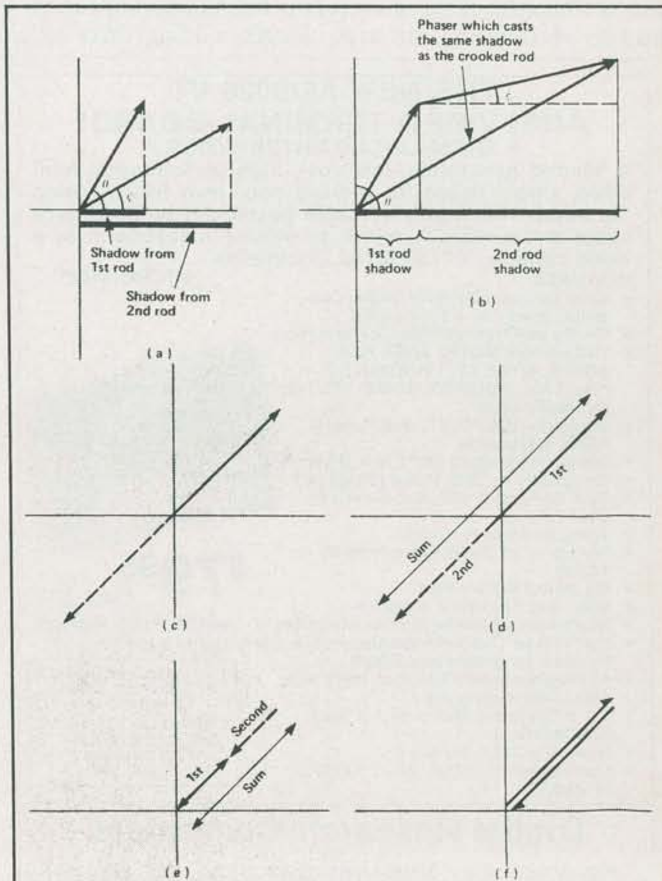

Fig. 5 (a). Adding two waves on the vector diagram, (b) the resultant wave, (c) adding out of phase waves, (d) the wrong result, (e) the correct answer, and (f) cancellation.

and it's the phase and amplitude of the waves that we want to investigate. This is where vector diagrams come into their own.

Just as several waves of different frequencies can be included in the same frequency spectrum diagram, waves at the same frequency but with different amplitudes and phase can be drawn on the same vector diagram (**Fig.5a**). Since both waves are at the same frequency, the two rods are locked together and rotate at the same speed. This gives an easy graphical way to find the sum of the sine waves of the same frequency.

To find the result of adding the two waves of Fig.5a, we want a rod which casts a shadow equal to the sum of the two individual shadows. A crooked rod that will do the trick can be made simply by nailing the two rods together, keeping their angles with the horizontal axis the same. The result is shown in **Fig. 5b**.

The final step is to notice that a straight rod from the origin to the tip of the crooked rod will cast the same shadow, so this is the vector representing the sum of the two original waves.

If you think about it for a moment, you'll see that it makes no difference which vector is drawn first when adding. If you try both ways on the same diagram, you'll end up with a parallelogram with the sum as one of the diagonals.

One thing to remember is that you must draw the vectors in the right direction. Take the extreme case of Fig.5c, for instance. The sum of these vectors is Fig.5d and not Fig.5e. If the two vectors had the same length, adding them would bring the end point right back to the origin (Fig.5f). This is the equivalent to saying that two sine waves of the same frequency and amplitude but 180° out of phase will cancel each other out when added.

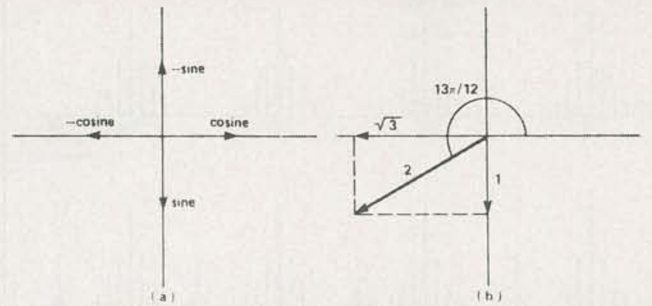

Fig. 6 (a). The vectors for sine, cosine, -sine and -cosine, and (b) expressing a wave as a sum of sine and cosine.

The vectors for a sine, cosine, -sine and -cosine are shown in **Fig.6a**. A sine wave of any phase you choose can be expressed as the sum of a sine and a cosine. Figure 6b shows a particular example:

$$2\cos(\omega t + \tfrac{13\pi}{12}) = \sin(\omega t) - \sqrt{3}\cos(\omega_5)$$

The phase angle 13pi/12 disappears. The Fourier series contain both sine and cosine terms to avoid the need for phase angles and this is how the trick is done. Another piece of the jigsaw falls into place. It's rather like resolving a force into two orthogonal components. ■

"Instead of just pulling in satellite TV signals, I'm going to pull in the whole satellite."