Bridge oscillators

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A bridge oscillator is one employing both positive and negative feedback at the inputs of an operational amplifier. By this means it is possible to imitate, using only resistors and capacitors, a high-Q LC oscillator which manifestly behaves in the same way, i.e with high frequency stability and low harmonic distortion; although neither of these properties can be associated with the often misnamed Wien bridge circuit shown in Fig. 1. The latter does not possess a bridge structure and only shows the properties described when modified to include negative feedback via a resistive arm as in Fig. 5. Nevertheless, it is the most common form and such oscillators are commercially available with a 10/1 variation in frequency per dial turn and overall coverage of 10^6/1 from, say, 1Hz to 1MHz. They possess two major advantages over LC oscillators, viz.;

(1) They have a wideband tuning capability. Thus, whereas in an LC oscillator a 10/1 variation in L or C produces a 3 16/1 variation in frequency, a 10/1 change in the R or C of a Wien bridge network produces a 10/1 change in the oscillation frequency

(2) At frequencies below several kilohertz the linear inductors required for an LC oscillator become unwieldy and expensive.

The two other common forms of RC bridge circuit, the balanced form of the symmetrical twin T, Fig. 2, and the bridged T, Fig. 3, possess similar advantages over LC networks. Furthermore, with the proviso that when used in an oscillator the same amplifier is used in all three RC configurations, they produce an output with lower harmonic distortion and better frequency stability than the Wien bridge

Bridge oscillators are not, however, exclusively of the RC type. Another form in common use is the Meacham bridge shown in Fig. 4, which employs a crystal instead of an RC network in the frequency selective arm. The existence of feedback via R_1 and R_2 increases the frequency stability and decreases the harmonic distortion compared to that of an oscillator in which the frequency is controlled by a crystal alone. The factor of improvement is of the order of the amplifier gain

Since the behaviour of each of the three RC bridge networks is very similar the earlier sections of the ensuing discussion will give a comparison (from the standpoint of harmonic distortion and frequency stability) of oscillators in which they are incorporated. This will be followed by an examination of the Meacham bridge. The final section will deal with the problem of amplitude stabilization

Harmonic distortion

In a self-starting oscillator the amplitude of oscillation increases until the amplifier gain × feedback fraction is unity. In the process, the active device will, for any real output swing, be operating over a large portion of its characteristics and consequently non-linearity distortion will be introduced. The harmonics thus produced may be treated as additional signals within the feedback loop and each harmonic will be acted upon by the factor

$$K_n = \frac{H_o}{H_a} = \frac{1}{(1-AB_n)}$$

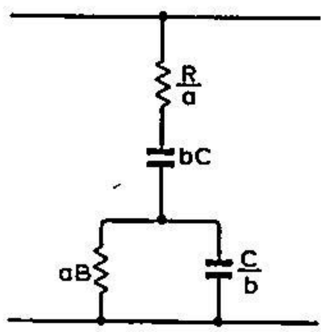


Fig. 1. Wien network, which becomes a bridge when completed by a resistive arm.

where H_o is the harmonic distortion in the oscillator, H_a the harmonic distortion in the amplifier, A the amplifier gain and B_n the feedback fraction for the nth harmonic. At the frequency of oscillation, f_o,

$$AB_o = 1 \text{ and } K_n = \frac{1}{A(B_o-B_n)},$$

i e $$|K_n| = \frac{1}{|A||(B_o-B_n)|} \quad (1)$$

Since it is desirable to produce an almost sinusoidal output then the harmonic distortion should be small. Each of the harmonics produced in the amplifier is modified by the factor $|K_n|$ and for a well designed network this factor should be as small as possible For a given amplifier this will occur when $B_o - B_n$ is large.

The more selective the feedback network

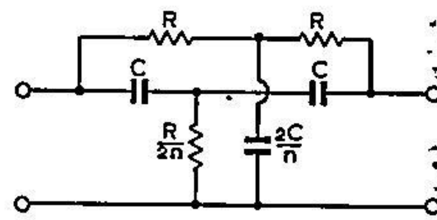
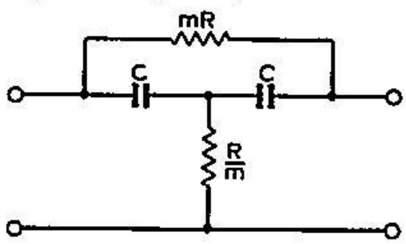


Fig 2. Twin T RC network.



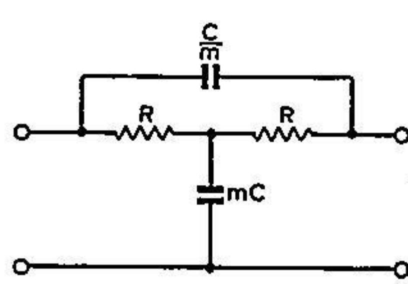


Fig. 3. Two forms of bridged T network, giving identical results.

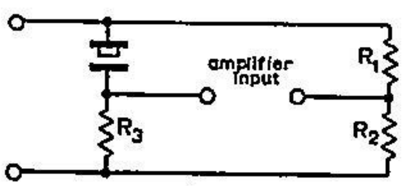


Fig. 4. Meacham bridge crystal network.

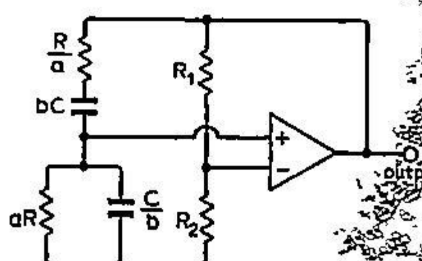


Fig. 5. General form of Wien oscillator.

the greater is the value of $|B_o - B_s|$ and, for a given degree of amplifier distortion, the more pure will be the output wave. From this point of view, therefore, the relative quality of two oscillators can be obtained by comparing their respective $|K_n|$ factors. Such a comparison will now be made for oscillators employing the feedback networks shown in Figs. 2, 3 and 5

Wien bridge. To incorporate the Wien bridge into an oscillator, the amplifier must have both an inverting and non-inverting input terminal. Since the frequency selective arm of the bridge has a maximum transmission at resonance then it is connected to the non-inverting terminal and negative feedback is applied via the resistive arm to the inverting input.

Assuming the amplifier input resistance is large compared to aR and R_2 the feedback fraction is given by Appendix 1 as

$$B = \frac{1}{(x + ju/ab)} - B_r \qquad (2)$$

where $x = 1 + \frac{1}{a^2} + \frac{1}{b^2}$,

$u = (\omega/\omega_o - \omega_o/\omega)$,

$B_r = \frac{R_2}{(R_2 + R_1)}$

Substituting for B_o and B_s in Eq. (1) gives $|K_n| = xab/A\{(x^2/u^2 + (1/ab)^2\}^{\frac{1}{2}}$ and this takes the values shown in Table 1 for the second and third harmonics.

Table 1

	K_2	K_3
$a=b=1$	6 69/A	4.5/A
$a=b=\sqrt{3}/2$	3 39/A	2 53/A

Obviously the larger is the value of A the smaller are the factors K_2 and K_3. Thus for the completely unbalanced bridge where there is no resistive feedback and the amplifier gain $= x$ then $K_2 = 6\ 69/x$. Using an amplifier gain of $100x$ reduces this factor a hundredfold but requires the introduction of resistive feedback in which the feedback ratio is controllable to within 1% of $1/x$

Twin T. The twin T, being a minimum transmission network, is connected in the negative feedback loop when employed in oscillator circuits The transfer response of the unloaded network is given by references 1, 2 and 3 as

$$\frac{v_o}{v_{in}} = \frac{1}{1 - 4j/u}$$

where $u = \omega/\omega_o - \omega_o/\omega$ and $\omega_o = n^{\frac{1}{2}}CR$. The sensitivity[3] of this function with respect to n, i.e. $d|(v_o/v_{in})|/dn$ is a maximum for $n = 1$ and this accounts for the fact that in oscillator circuits it is more generally seen in the form shown in Fig. 6. For this reason attention will be restricted to this type of network in which

$$B_{tt} = B_r - \frac{1}{1 - 4j/u}$$

where $B_r = R_2/(R_1 + R_2)$. Making the relevant substitutions in Eq. 1 gives the results shown in Table 2.

Table 2

K_2	K_3
8.85/A	1.81/A

From a comparison of Tables 1 and 2 it would seem, therefore, that the twin T oscillator shows lower harmonic distortion. Unfortunately, variation of the oscillator frequency requires that three elements be altered simultaneously In general, therefore, twin T oscillators are only used in cases of fixed frequency operation

Bridged T. The bridged T network may take either of the forms shown in Fig. 3, there being no difference in their behaviour. Since the circuit is a minimum transmission network it is used in the negative feedback arm as in Fig. 7 to provide the frequency selectivity necessary in an oscillator.

The feedback fraction B_{bt} is given by:[1]

$$B_{bt} = B_r - \frac{(m - 2j/u)}{(m - j(m^2+2)/u}$$

where $B_r = R_2/(R_1 + R_2)$ and $u = (\omega/\omega_o - \omega_o/\omega)$ Substituting for B_o and B_s in Eq. 1 gives

$$|K_n| = \frac{1}{m^3 A}(2 + m^2)\left(m^2 + \frac{(m^2+2)^2}{u^2}\right)^{\frac{1}{2}}$$

which leads to the results shown in Table 3, where the factors $m = 2.5$ and $m = 3$, since they approximately correspond to the values for m where $|K_2|$ $(m = 2.5)$ and $|K_3|$ $(m = 3)$ are at their minimum

Table 3

m	K_2	K_3
1	6.69/A	6.3/A
2.5	2.55/A	2.14/A
3	3.22/A	2.08/A

Frequency stabilization

The frequency of oscillation is identically that for which the phase shift round the loop is zero. If some parameter of the amplifier should change and produce a phase shift then the oscillation frequency will change to re-establish the zero phase condition. For example, if the change in the amplifier produces a phase shift of, say, 1° and if the phase shift of the bridge network in the neighbourhood of the oscillation frequency is −10° per 1000Hz then there will be an increase in the oscillation frequency of 100Hz. Obviously the greater the rate of change of phase with frequency of the bridge the greater will be its frequency stability.

The 100Hz change in oscillator frequency due to the parameter change in the amplifier was calculated on the basis that

$$\Delta f = \frac{\Delta\phi}{S_f'}$$

where $\Delta f = 100\text{Hz}$, $\Delta\phi = 1° =$ phase shift in amplifier, and $S_f' = d\phi/df = 0.01°/\text{Hz}$, which is the rate of change of phase with frequency of the frequency-determining network. From a practical point of view it is much more important to know the fractional change in frequency i e $\Delta f/f_o$. Consequently, in making a comparison of the frequency stability of oscillation it is usual to use a stability factor $S_f = f_o S_f'$ of the frequency selective network as the basis of the comparison. A phase shift of $\Delta\phi$ in the amplifier then produces a fractional change of $\Delta f/\Delta f_o = \Delta\phi/S_f$.

As shown in Appendix 2, S_f for each of the three networks so far considered has a maximum value of $0.5A$ for the twin T, $0.47A$ for the bridged T and $0.22A$ for the Wien bridge. When S_f is plotted as a function of f/f_o, for A = constant = 10, the curves take on the form shown in Fig. 8. Increasing A produces curves with a similar shape but with a higher peak value and a faster rate of fall off. It can be seen from these curves that, again, the twin T, inherently, gives the best oscillator.

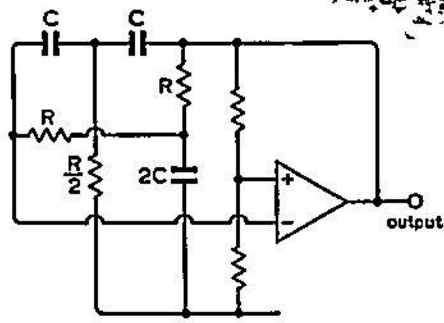


Fig. 6. Twin T oscillator.

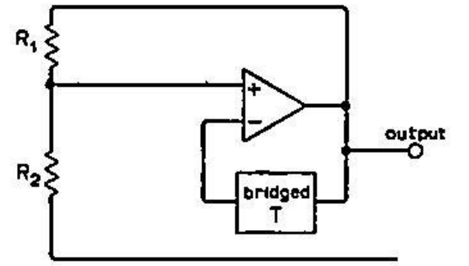


Fig. 7. Bridged T oscillator.

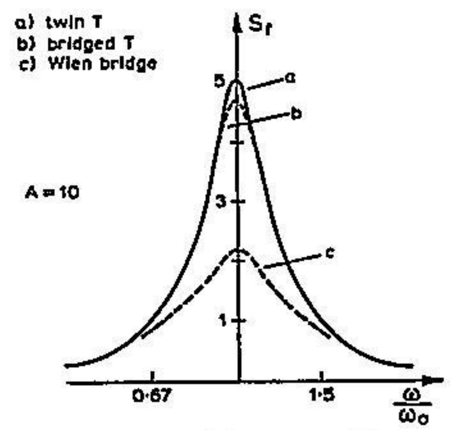


Fig. 8. Variation of frequency stability factor with frequency.

Meacham bridge. The Meacham bridge oscillator shown in Fig. 9 combines the properties of a high-Q LC circuit and an almost balanced bridge network to effect an extremely precise frequency standard, with the highest stability of any cricuit yet devised. The positive feedback is frequency dependent attaining a maximum at the series resonant frequency of the crystal. Assuming there is no phase shift in the amplifier, oscillation takes place at this frequency

Using the equivalent series model of the crystal the feedback fraction B_{mb} is given by:

$$B_{mb} = \frac{R_3}{(R_3+r+j\omega L+1/j\omega C)} - B_r$$

where $B_r = R_2/(R_1+R_2)$. Substituting $N = r/R_3 = 1/(LC)^{\frac{1}{2}}$ and $Q = L/r$ this becomes

$$B_{mb} = \frac{1}{(N+1)+jNQu} - B_r.$$

This equation is of the same form as Eq. (2) for the Wien bridge, the factors $N+1$ and QN being analogous to x and $1/ab$ respectively. By analogy it follows that the factor K_n for the Meacham bridge may be obtained using the earlier results and replacing x and ab by their analogies. Hence

$$K_n = \frac{(N+1)}{AQN}\left\{\frac{(N+1)^2}{u^2}+Q^2N^2\right\}^{\frac{1}{2}}$$

For all harmonics $u = (\omega/\omega_o - \omega_o/\omega) > 1$ so that $(N+1)/u^2 > (N+1)$ Furthermore, since Q is very large (typically 20,000) and providing N is not too much smaller than unity this approximates to

$$K_n = \frac{(N+1)}{NA}$$

which is completely independent of the resistive side of the bridge.

In a similar manner by substituting for $x = (N+1)$ and $QN = 1/ab$ the expression for the frequency stability factor S_f at the frequency of oscillation is (Appendix 2)

$$S_f = -\frac{(2QNA)}{(N+1)^2}$$

This function is geometrically symmetrical about $N = 1$ and has a maximum value of $-QA/2$ at this point.

Amplitude stabilization

The amplitude stability of an oscillator is the sensitivity of the oscillation amplitude V_o to variations in temperature, supply voltage etc. In particular, the amplitude stability factor is defined as

$$S\mu = \frac{\Delta V_o/V_o}{\Delta\mu/\mu}$$

where μ is the parameter within the oscillator which is subject to variation. Obviously, the lower the value of $S\mu$ the more stable will be the output amplitude and frequently amplitude control elements are used to provide the necessary stabilization.

To demonstrate the function of these control elements imagine that the feedback resistor R_2 of the Wien bridge network increases with the voltage across it. If the output amplitude should increase then R_2 and hence the feedback fraction $R_2/(R_1+R_2)$ will increase This reduces the voltage fed back and causes a decrease in output.

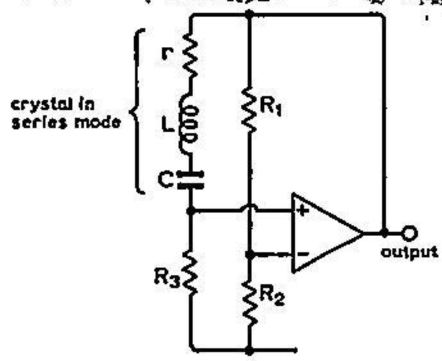


Fig. 9. Meacham bridge oscillator, showing the crystal as a tuned circuit.

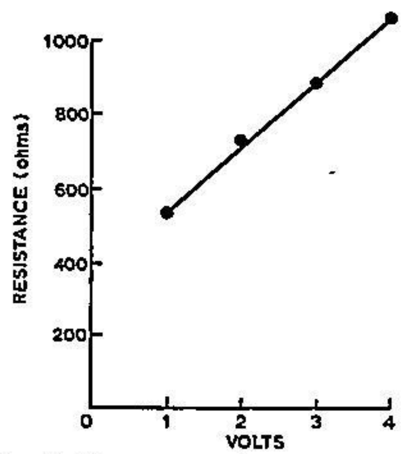


Fig. 10. Characteristics of a 6V, 0.36W tungsten-filament bulb.

One form the stabilization element may take is a tungsten filament lamp. Typically these lamps have a characteristic of the form shown in Fig. 10 in which case over a wide voltage range the resistance R_2 is given by:

$$R_2 = R_o + KV_r;\ V_r = \text{volts across lamp.}$$

Under oscillatory conditions

$$B_r = \frac{R_2}{R_1+R_2} = \left(\frac{1}{3}-\frac{1}{A}\right) \text{ and } B_rV_o = V_r$$

therefore

$$dB_r = \frac{KR_1}{(R_o+KV_r+R_1)^2}\,dV_r = \frac{dA}{A^2}$$

But

$$\frac{R_1}{R_o+KV_r+R_1} = 1 - B_r = \frac{2}{3}+\frac{1}{A}$$

$$\simeq \frac{2}{3} \text{ for large } A.$$

Hence

$$\frac{dA}{A} \simeq \frac{2KA}{3(R_o+R_1+KV_r)^2}\,dV_r$$

and

$$S_A = \frac{dA/A}{dV_o/V_o} = \frac{2K}{3A(R_1+R_o+KV_r)}\frac{dV_o}{dV_r}V_o.$$

Since $V_r = 1/3V_o$ then $dV_r = 1/3dV_o$ and

$$S_A = \frac{2KV_o}{9A(R_1+R+K/3V_o)}$$

In the limit as K tends to infinity then S_A tends to a maximum of $2/3A$.

The amplitude control element may take many different forms. In the Meacham bridge circuit the resistor R_2 needs to increase as V_o increases. Practically this may be accomplished using an f.e.t.[4,5]. The output voltage is rectified and used to control the drain-source resistance of the f.e.t. Quantitively the stabilization factor may be assessed in a manner similar to that used for the Wien bridge.

Appendix 1

For the Wien bridge the voltage, v_f, fed back is given by:

$$v_f = \frac{\dfrac{aR}{(aR+b/j\omega C)}\cdot\dfrac{b}{j\omega C}}{\dfrac{1}{j\omega bC}+\dfrac{R}{a}+\dfrac{aR}{(aR+b/j\omega C)}\dfrac{b}{j\omega C}} - B_r$$

$$= \frac{abR}{(a/b)R+(b/a)R+abR+1/j\omega C+j\omega CR^2} - B_r$$

$$= \frac{1}{\left(1+1/a^2+1/b^2+\dfrac{j\omega CR-j/\omega CR}{ab}\right)} - B_r$$

$$= \frac{1}{x+ju/ab} - B_r. \quad \text{(A1)}$$

where $x = 1+1/a^2+1/b^2$, $u = (\omega/\omega_o - \omega_o/\omega)$ and $\omega_o = 1/CR$.

Appendix 2

From Appendix 1 it follows that at the frequency of oscillation

$$B_{wb} = \frac{1}{x} - B_r.$$

Also, at this frequency the loop gain $AB_{wb} = 1$ so that $B_{wb} = 1/A$. It follows that $B_r = 1/x - 1/A$ Substituting for this factor in Eq A1 gives

$$B_{wb} = \frac{\dfrac{x}{A}+\dfrac{ju}{ab}(1/A-1/x)}{x+ju/ab} \quad \text{(A2)}$$

The loop gain $AB_{wb} = |AB_{wb}|\ \phi$ where ϕ = phase shift. Assuming there is no phase shift in the amplifier, $\phi = \phi_{wb}$ (phase shift of B_{wb}). Hence from Eq. A2

$$\phi_{wb} = \tan^{-1}\frac{Au}{xab}(1/A-1/x) - \tan^{-1}(u/xab)$$

$$S_f = f_o\frac{d\phi}{d\omega} = \left[\frac{A}{xab}\left\{\frac{1}{A}-\frac{1}{x}\right\} \times \left\{\frac{1}{1+\{Au/xab(1/A-1/x)\}^2}\right\} - \left(\frac{1}{xab}\right)\left(\frac{1}{1+u^2/x^2a^2b^2}\right)\right] \times [1+\omega_o^2/\omega^2]. \quad \text{(A3)}$$

When plotted as a function of ω/ω_o, S_f takes a maximum value when this ratio is unity. Under this condition Eq. A3 simplifies to give

$$S_f = \frac{2}{xab}(-A/x)$$

$$= -0.22A \text{ when } a = b = 1.$$

Similarly for the twin T

$$B_r = 1/A$$

$$B_a = \frac{(1/A-1)-4j/Au}{(1-4j/u)}$$

$$\phi_a = \tan^{-1}(4/u(A-1))+\tan^{-1}(4/u)$$

$$S_f = \{1+\omega_o^2/\omega^2\}\left\{\frac{-4}{u^2(A-1)+16/(A-1)}\right\}$$

$$-\frac{4}{u^2+16}.$$

At $\omega = \omega_o$, $u = 0$ and $S_f = -A/2$.

Finally, for the bridged T

$$B_r = 1/A+2/(m^2+2)$$

$$B_{bt} = \frac{\{m/A-m^3/(m^2+2)\}-j(m^2+2)/Au}{m-j(m^2+2)/u}$$

$$\phi_{bt} = \tan^{-1}(m^2+2)/Au\alpha$$

$$+\tan^{-1}(m^2+2)/mu,$$

$$\alpha = m/A-m^3/(m^2+2)$$

$$S_f = \{1+\omega_o^2/\omega^2\}\left\{\frac{(m^2+2)}{A\alpha}\right\}$$

$$\times\left\{\frac{1}{u^2+(m^2+2)^2/A^2\alpha^2}\right\}$$

$$-\frac{(m^2+2)/m}{u^2+\{(m^2+2)/m\}^2}.$$

Substituting $m = 2.5$ and $\omega = \omega_o$ (i.e. $u = 0$) gives $S_f = -0.47A$.

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2. Girling, F. E. J. and E. F. Good, *Wireless World*, Oct. 1969, p. 461
3. Valley, G. E. and H. Wallman, "Vacuum Tube Amplifiers", McGraw-Hill, New York (1948), Chapter 10
4. Strauss, L., "Wave Generation and Shaping", Second Ed., McGraw-Hill (1970), Chapter 16.
5. Clarke, K. K. and D. T. Hess, "Communication Circuits", Addison-Wesley, 1971, Chapter 6.

Books Received

Radio Wave Propagation, by Armel Picquerand, is suitable for radio engineers who want a better understanding of the phenomena. The initial chapters of the book summarize the present state of our knowledge of wave propagation as far as it concerns the telecommunications engineer. Following chapters give graphs which can be used for the calculation of the principal parameters of communication circuits. An appendix gives a brief review of those concepts of mathematical physics required for the comprehension of the theoretical discussions in this book. Price £10. Pp. 343. Macmillan, 4 Little Essex Street, London WC2 RLF.

Electrotechnology Basic Theory and Circuit Calculations for Electrical Engineers, by M. G. Say, starts with a chapter on units and physical quantities. A second chapter deals with the physical nature of electric charge and conduction and then explains the effects in practical terms. A third chapter deals with network analysis, starting with basic circuit parameters and outlining some of the short-cut techniques devised to solve particular groups of problems. The book concludes with a chapter on special techniques of circuit analysis. Price £1.70. Pp. 176. Newnes-Butterworths Ltd, Borough Green, Sevenoaks, Kent TN15 8PH.

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Slow Scan Television Handbook is suitable for the amateur interested in s.s.t.v. After a brief historical introduction the book explains the basic principles involved together with some popular circuits used in s.s.t.v. A chapter on monitors gives several full circuit diagrams and parts lists. Subsequent chapters deal with flying spot scanners, live vidicon cameras, colour equipment and applications of audio filters. The book concludes with chapters on test gear and commercial equipment. Price £2 plus 20p postage and packing. Pp. 248. British Amateur Television Club, c/o "White Orchard", 64 Showell Lane, Penn, Wolverhampton, Staffs WV4 4TT.

Questions and Answers Integrated Circuits, by R. G. Hibberd, is a pocket book aimed at helping students or technicians understand i.cs. The sections in the book are: basic aspects of i.cs., i.c. technology, digital, linear and m.o.s. i.cs., m.s.i. and l.s.i. and applications. Price 75p. Pp. 96. Butterworth & Co Ltd, Borough Green, Sevenoaks, Kent TN15 8PH.

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