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Subaudio Tunable Amplifier

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ONE METHOD of analyzing low-frequency components of complex waveforms is to pass them through a tunable, narrow-band amplifier. Design and performance of such an amplifier, tunable from 0.5 to 100 cps, is presented.

Amplifier Unit

The unit in Fig. 1 consists of a mixing circuit, inverting operational d-c amplifier and twin-tee tuning element. There are two negative feedback paths around the d-c amplifier: R_1 reduces gain to the desired maximum and provides stability, and the twin-tee network degenerates all but the desired frequency.

A 5-megohm potentiometer at the output of the twin-tee allows Q to be reduced without affecting overall gain. Maximum Q available is determined by R_1.

The resistance arms of the twin-tee consist of three ganged 10-turn potentiometers that are trimmed and have their shaft couplings adjusted until a resistance bridge measurement indicated that they tracked with less than 0.2 percent error from 5,000 to 100,000 ohms. The three associated capacitors were measured on an impedance bridge and matched in a deviation bridge to within 0.1 percent. As a result the twin-tee alone exhibits a notch of more than 45 db from 0.5 to 20 cps and more than 30 db through 100 cps.

Single-knob control provides continuous tuning from 0.5 to 100 cps with a Q of 50 from 0.5 to 20 cps. Above 20 cps, gain and Q decrease because of tracking errors in the twin-tee. However, a Q of over 100 can easily be reached by changing the value of R_1 without danger of oscillation. Gain is about 200. Input level must be kept low enough to prevent distortion from overdrive (in this case 0.1 v).

Time Constant

At these low frequencies the effect of the time constant is very pronounced. At one cps, input can be removed and output will take about a half minute to die down to noise level.

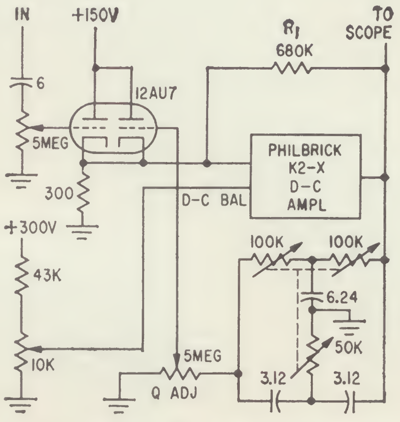

FIG. 1—Commercial d-c amplifier with twin-tee feedback added tunes from 0.5 to 100 cps

By experiment, the constant relating time constant (TC) and Q was found to be

$$k = (TC)\ f/Q \approx 0.32 \qquad (1)$$

The time constant was measured at frequencies below 2 cps by abruptly removing the signal and measuring with a stop watch the time for the output to fall to one-half maximum. After output had fallen to noise level, the signal was abruptly applied and the time measured for output to reach this half amplitude point again. (At one cps, this time can be 6 or 7 sec.). Since universal time constant curves cross at half amplitude for 0.73 time constant, either time as measured above can be divided by 0.73 to give the circuit time constant.

For higher frequencies short pulses were applied at a prf about one-twentieth the frequency of the twin-tee. An exponentially decaying sine wave appeared at the output. On the scope, time taken for amplitude to fall to $1/\epsilon$ of maximum was measured. This value is a time constant by definition.

Measuring Q

The Q was found by measuring gain (A) of the d-c amplifier and feedback loop and applying the formula $Q = A/4$. Another method was to vary input frequency until output amplitude fell to 0.707 of maximum, since $Q = f_o/(f_2 - f_1)$, where f_o is center frequency, f_2 is upper frequency at 0.707 maximum amplitude and f_1 is lower frequency at 0.707 maximum amplitude.

The value of $k = 0.32$ obtained by inserting measured values of TC and Q in Eq. 1 agrees with the following mathematical derivation. The solution of the differential equation for a high Q-L-C-R circuit yields:

$$E_1 = E_o \epsilon^{-Rt_1/2L} \qquad (2)$$

where E_1 is amplitude at time t_1 and E_o is amplitude at time t_o. Since $Q = 2\pi fL/R$,

$$L/R = Q/2\pi f \qquad (3)$$

Substitution of Eq. 3 in Eq. 2 gives:

$$E_1 = E_o \epsilon^{-\pi f t_1/Q} \qquad (4)$$

If t_1 is selected equal to TC, $E_1 = E_o/\epsilon$, so that Eq. 4 becomes $E_o/\epsilon = E_o\epsilon^{-\pi f(TC)/Q}$. Solving: $f(TC)/Q = 1/\pi \approx 0.32$.

If pulses are applied to the circuit with a prf of $1/t_1$, $Q = -\pi f/(\text{prf}) \ln (E_1/E_o)$. Since E_1 and E_o can be measured on a scope and the other values are known, Q can be calculated. This was also in agreement with the other methods used.

If random noise is applied to the circuit, a sine wave output appears at the frequency to which the twin-tee is tuned, since the unit accepts only that frequency component of the noise. A one-cps square wave was analyzed for frequency components up to its 15th harmonic (down 23 db from the fundamental). The two abrupt changes per second contained in a one-cps square wave cause a ringing at 2 cps that is of sufficient amplitude to hinder measurements above the 15th harmonic, although harmonics as high as the 50th can be seen.

It appears feasible to use this type circuit with higher Q's, lower frequencies, greater dynamic range or wider tuning range if desired.

The background information for this paper was obtained from Naval Research Lab. report 4444 and from unpublished papers by the author.

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